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Attitude Control of a Flexible Spacecraft

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A parameter optimization algorithm for designing optimal low-order controllers for high-order systems is applied to the title problem where many vibration modes are excited by the control torque. These low-order controllers are compared with the full-order optimal controller and found to be less sensitive to modeling errors and to provide near optimal attitude regulation.

Introduction

OFTEN several vibration modes must be included to model the dynamics of a flexible spacecraft with sufficient accuracy to design a satisfactory control system.

A controller designed to minimize a weighted sum of mean square output and mean square input (the performance index, PI) must have the same dimension as the flexible spacecraft model, since all of the estimated state variables must be fed back to the controls. Not only is this unnecessarily complex, but the controller may be very sensitive to modeling errors, since the vibration modes are often stabilized by placing controller zeros and poles very close to the spacecraft vibration poles. A small deviation in the actual frequencies from the model frequencies may cause the controlled system to go unstable.

The same PI is used here, but the order of the controller is constrained to be less than the order of the spacecraft model. Gradients of the PI with respect to the parameters of this reduced order controller are determined by solving a linear matrix Lyapunov equation. A standard nonlinear programming algorithm is used with these gradients to find the parameters that minimize the PI.

Parameter optimization methods for finding low-order controllers have been presented by various authors.[1-4] The algorithm described below follows Kwakernaak and Sivan.[4] This algorithm is used here to find reduced-order controllers for a spacecraft with large flexible solar panels. These controllers are found to be less sensitive to spacecraft modeling errors and produce only modest increases in PI relative to the full-order controller.

Problem Formulation

The system to be controlled is described by the linear equations

$$\dot{x} = Fx + Gu + G_2 w \qquad y = Hx + v \tag{1}$$

The vectors w and v are modeled as independent zero-mean white noise with spectral densities R_w and R_v, respectively. Some or all of the measurements may be modeled as containing colored instead of white noise. This would be appropriate if a sensor's output is prefiltered before being available for feedback. In that case, elements of the noise vector v would be zero.

For the system of Eq. (1), a lower-order linear time invariant controller is desired which minimizes the performance index

$$J = \mathrm{tr}[Q_1 X(\infty)] + \mathrm{tr}[Q_2 U(\infty)] \tag{2}$$

subject to the constraints that

$$\dot{x}_c = Ax_c + By \qquad u = Cx_c + Dy \tag{3}$$

where $X(\infty)$ is the steady-state covariance of the state vector x, $U(\infty)$ is the steady-state covariance of the control u, and x_c is the controller state vector.

The matrix D allows a measurement to be fed directly to the input u. In order to keep the control covariance bounded, Dv must be the zero vector, although D can be nonzero if there is an output which is not contaminated with white noise.

Because an invertible linear transformation of the controller states does not change the performance index, Eq. (2) cannot have a unique minimum if all the parameters of the matrices A, B, C, and D are treated as independent variables. Only the controller transfer functions from the outputs y to the inputs u are of importance.

Denery[5] has shown that only $m(n_0+n_c)$ parameters are necessary to specify the controller transfer functions when no direct feedthrough is allowed. Here m is the controller order, n_0 is the number of outputs, and n_c is the number of controls. The addition of a full direct feedthrough matrix D would bring the total number of free parameters to $m(n_0+n_c)+n_0 n_c$.

Augmented Equations

By defining an augmented state vector

$$x_a = \begin{bmatrix} x \\ \hline x_c \end{bmatrix} \tag{4}$$

the dynamic equations, Eqs. (1) and (3), can be combined as

$$\dot{x}_a = \left[\begin{array}{c|c} F+GDH & GC \\ \hline BH & A \end{array}\right] x_a + \left[\begin{array}{c|c} G_2 & GD \\ \hline 0 & B \end{array}\right] \begin{bmatrix} w \\ v \end{bmatrix} = F_a x_a + G_a n_a \tag{5}$$

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Only controllers for which the matrix F_a is stable are of interest. When F_a is stable, the covariance of x_a will reach the steady state value which satisfies the Lyapunov equation

$$F_a X_a + X_a F_a^T = -G_a R_a G_a^T \tag{6}$$

where

$$X_a = E(x_a x_a^T) \tag{7}$$

and

$$R_a = \left[\begin{array}{c|c} R_w & 0 \\ \hline 0 & R_v \end{array}\right] \tag{8}$$

In terms of the augmented state covariance, the performance index is

$$J = \mathrm{tr} \left[\begin{array}{c|c} Q_1 + H^T D^T Q_2 DH & H^T D^T Q_2 C \\ \hline C^T Q_2 DH & C^T Q_2 C \end{array}\right] X_a$$

$$= \mathrm{tr}(Q_a X_q) \tag{9}$$

Numerical Solution

The controller matrices A, B, C, and D determine the performance index nonlinearly through Eqs. (6) and (9). A local minimum of the performance index with respect to the parameters of these matrices can be found with a gradient method.

Kwakernaak and Sivan[4] have shown that the gradient can be computed easily by using a matrix of Lagrange multipliers, Λ, which satisfies

$$F_a^T \Lambda + \Lambda F_a = -Q_a \tag{10}$$

This is a Lyapunov equation adjoint to Eq. (6).

Let P be a vector whose components are the independent parameters of the matrices A, B, C, and D. With Λ defined by Eq. (10), the partial derivative of the performance index can be written as

$$\frac{\partial J}{\partial P_i} = \mathrm{tr}\left\{\frac{\partial Q_a}{\partial P_i} X_a + 2\left[\frac{\partial G_a}{\partial P_i} R_a G_a^T \Lambda + \frac{\partial F_a}{\partial P_i} X_a \Lambda\right]\right\} \tag{11}$$

The partial derivatives of the matrices Q_a, G_a, and F_a are sparse matrices. For example, if $P_i = a_{jk}$, a parameter of the controller matrix A, then $\partial Q_a/\partial P_i$ and $\partial G_a/\partial P_i$ are null matrices, while $\partial F_a/\partial P_i$ has a single "1" as its $(n+j,\ n+k)$ element.

Most of the computation required to compute the gradient of the performance index is involved with the solution of the Lyapunov equations, Eqs. (6) and (10). These equations should be solved with an efficient direct method such as the Bartels-Steward algorithm described in Ref. 6. Because Eqs. (6) and (10) contain the same matrix F_a, direct solution of these equations can be much faster than an iterative algorithm. Once the matrices X_a and Λ have been calculated, Eq. (11) can be solved rapidly for the partial derivative of the performance index with respect to each of the controller parameters. A more detailed description of the algorithm is given in Ref. 7.

Control of an Elastic Spacecraft

The algorithm described above was used to find reduced-order controllers for a solar electric propulsion spacecraft. Attitude control torques can be applied to this spacecraft by differential gimbaling of six constant-force ion propulsion thrusters. It was designed for electric propulsion during deep-space operation and requires large lightweight solar panels. Since the vibration frequencies of the solar panels are quite low, the interaction of the attitude control system with the vibration modes has been a major concern.

Larson and Likins[8] studied the effects of using truncated low-order models to design the attitude control system. They concluded that this method of controller design could produce severe consequences. To compensate for model errors, Skelton and Likins[9] suggest the use of an "orthogonal filter." The orthogonal filter is designed to approximate an unknown state estimation error with orthogonal functions of time. They presented an example where the addition of an orthogonal filter to the state estimator improved the attitude regulation in the presence of unmodeled constant disturbances.

Optimal reduced-order controllers for this spacecraft are presented here. A fairly accurate full-order spacecraft model must be available to design these controllers, but it is shown that the low-order controllers are not too sensitive to errors in the model. Constant disturbances could be accommodated with integral control, but only zero-mean disturbances will be considered.

System Equations

Larson and Likins[8] and Skelton and Likins[9] considered the pitch attitude control of the spacecraft and ignored the small coupling with yaw and roll attitude. The spacecraft model and parameter values given in their papers will be used here.

The spacecraft was modeled using a system of hybrid coordinates, consisting of an attitude angle for the central body, θ, and model coordinates for the solar panels, η. From the paper by Larson and Likins,[9] the equations of motion may be written as

$$J_\theta \ddot{\theta} - \delta^T \ddot{\eta} = b_r u \tag{12}$$

$$I_J(\ddot{\eta} + \zeta\sigma\dot{\eta} + \sigma^2\eta) - \delta\ddot{\theta} = 0 \tag{13}$$

where J_θ is the moment of inertia of the undeformed spacecraft about the pitch axis, δ is the vector of modal "coupling scalars," b_r is the control torque about pitch axis per unit u, I_j is the unit matrix with dimensions of moment of inertia, ζ is the diagonal matrix of modal damping ratios, and σ is the diagonal matrix of modal frequencies.

These equations can be expressed as the single matrix equation

$$M\begin{bmatrix}\ddot{\theta}\\ \ddot{\eta}\end{bmatrix} + D\begin{bmatrix}\dot{\theta}\\ \dot{\eta}\end{bmatrix} + K\begin{bmatrix}\theta\\ \eta\end{bmatrix} = gu \tag{14}$$

with

$$M \equiv \begin{bmatrix} J_\theta & -\delta^T \\ -\delta & I_J \end{bmatrix} \tag{15}$$

$$D \equiv \begin{bmatrix} 0 & 0 \\ 0 & -2\zeta\sigma \end{bmatrix} \tag{16}$$

$$K \equiv \begin{bmatrix} 0 & 0 \\ 0 & -\sigma^2 \end{bmatrix} \tag{17}$$

$$g \equiv \begin{bmatrix} b_r \\ 0 \end{bmatrix} \tag{18}$$

Parameter Values

The torque capacity of the spacecraft's ion engines is measured by b_r. The high-thrust verison of the spacecraft ($b_r = -11.2$ ft-lb) will be considered in this paper.

The attitude angle θ can be measured by a star sensor.

Table 1 Values of spacecraft parameters

Mode	σ_{ii}, s^{-1}	δ_i, slug-ft^2
1	0.4398	172.16
2	0.6686	−4.95
3	0.9802	−41.33
4	1.433	2.48
5	1.486	−1.63
6	1.505	12.22

J_θ = 33,353 slug-ft^2, $\zeta_{ii}=0.005$
b_r = −11.2 ft-lb, $R_w = 1.0\times10^{-6}$ rad^{2}s
R_v = 2.1×10^{-8} rad^{2}s

Skelton and Likins modeled the sensor output as

$$y=300\,\theta+v \tag{19}$$

The measurement noise v will be modeled as zero-mean white noise with covariance R_v.

Disturbances to the spacecraft will also be modeled as zero-mean white noise. Larson and Likins added a disturbance w to the control input u. The covariance R_w of this disturbance was based on an estimated uncertainty in assigning motor gimbal angles.

The values of the appendage modal frequencies σ_{ii}, the modal coupling scalars δ_i, and the other parameter values for this problem are given in Table 1.

Poles and Zeros

Six appendage vibration modes were included in the system model. These were the only modes which have significant coupling with rotation of the spacecraft about the pitch axis. With these modes and the rigid-body mode, the state equation [Eq. (21)] is fourteenth order.

Figure 1 shows the poles and zeros of the transfer function from u to θ. The zeros of the transfer function correspond to the eigenvalues of the *appendage-alone* vibration modes; the appendages vibrate at their natural frequencies, while the input supplies the torque necessary to keep the central body from rotating.

From Fig. 1, it can be seen that three of the *spacecraft* vibration modes have nearly the same natural frequency as the appendage-alone vibration modes; this is a result of appendage modes 2, 4, and 5 having very small coupling with rotation about the pitch axis.

Alternating Poles and Zeros and Classical Lead Compensation

If the three modes 2, 4, and 5 are neglected [these poles are nearly cancelled by zeros in the transfer function $\theta(s)/u(s)$], the remaining poles alternate with zeros along the imaginary axis. Such a system can be stabilized by using simple lead compensation:

$$\frac{u(s)}{y(s)}=K\frac{s+a}{s+b} \qquad (a<b) \tag{20}$$

Using such a compensation implicitly assumes negligible measurement noise, i.e., $v=0$ in Eq. (19). Figure 2 shows a root locus vs the gain K for $a=0.1$ s^{-1}, $b=0.6$ s^{-1}. The compensator pole and zero are labeled with a "C." Note that all closed-loop poles are stable and all but the highest frequency poles are moved closer to zeros. (Recall that the closer a pole is to a zero, the less that mode appears in the output.)

It is known that a vibrating distributed-parameter system without damping has an infinite number of alternating poles and zeros in the transfer function from torque (or force) to angular (or linear) position if the actuator and sensor are *collocated*. Simple lead compensation moves *all* the poles of such a system toward a neighboring zero along a path in the left-half plane; this is easily demonstrated using root locus concepts. Simple position-rate feedback (compensator zero at $s=0$) cancels one of the two poles at $s=0$ and the other pole moves along the negative real axis as the gain is increased; all other poles move toward a neighboring zero in a stable manner.

Optimal Control Problem

By defining the state vector as

$$x=[\theta\ \eta^T\ \dot\theta\ \dot\eta^T]^T \tag{21}$$

the equations of motion can be placed in state variable form with

$$F=\left[\begin{array}{c|c} 0 & I \\ \hline M^{-1}D & M^{-1}K\end{array}\right] \qquad G=G_2=\left[\begin{array}{c} 0 \\ \hline M^{-1}g\end{array}\right]$$

$$H=[300,\quad 0\,|\,0] \tag{22}$$

The control input u was chosen to minimize the quadratic performance index

$$J=(X_\theta+X_{\dot\theta})10^4+U \tag{23}$$

Here, X_θ and $X_{\dot\theta}$ are the steady-state covariance of angle θ and its derivative $\dot\theta$ (time in s). The mission requirements were

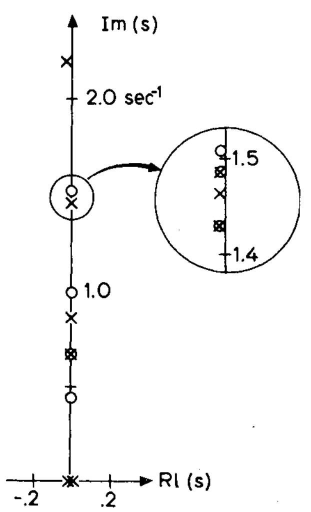

Fig. 1 Poles and zeros of transfer functions from u to θ.

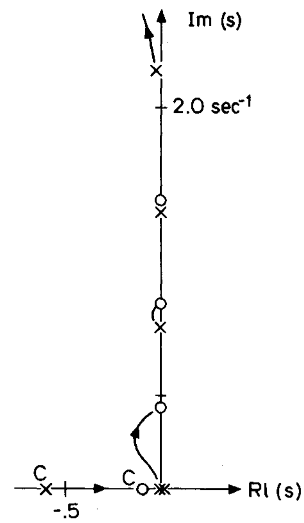

Fig. 2 Root locus vs overall gain with classical first-order lead compensator, used with eighth-order model of spacecraft.

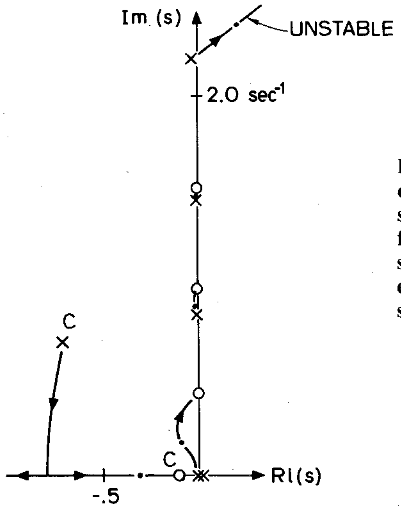

Fig. 3 Root locus vs overall gain with optimal second-order controller for second-order "rigid" spacecraft, used with eighth-order model of spacecraft.

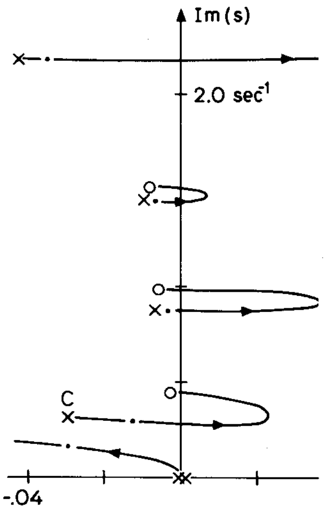

Fig. 5 Root locus vs overall gain with optimal second-order controller for sixth-order model of spacecraft, used with eighth-order model of spacecraft; abscissa scale enlarged.

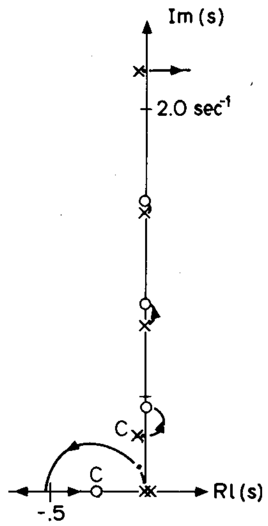

Fig. 4 Root locus vs overall gain with optimal second-order controller for sixth-order model of spacecraft, used with eight-order model of spacecraft.

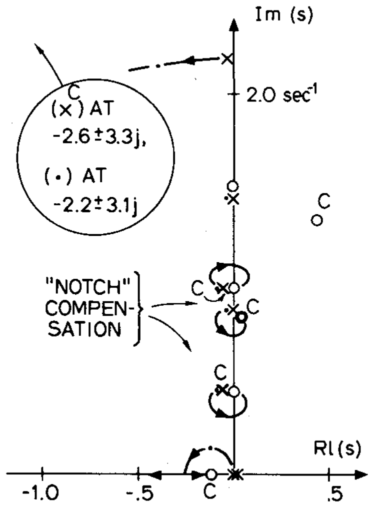

Fig. 6 Root locus vs overall gain with optimal sixth-order controller for sixth-order model of spacecraft, used with eight-order model of spacecraft.

expressed wholly in terms of θ and $\dot{\theta}$. Thus, Larson and Likins chose not to place a cost on appendage vibrations directly, but only through their effect on the central body attitude angle θ.

If the spacecraft were modeled as *rigid*, then the optimal controller would have the transfer function

$$\frac{u(s)}{y(s)} = \frac{2.19(s+0.107)}{s+0.72 \pm j0.72} \tag{24}$$

Note that this controller has two poles (complex conjugates) and one zero; there must be at least one more pole than zero to prevent the white noise in the measurement from passing directly through to the output. Using this controller with the fourteenth-order spacecraft model results in an unstable system. This can be seen from the root locus shown in Fig. 3. As in Fig. 2, the poles near $s = j0.668, j1.43$ and $j1.49$ are not shown because they are nearly cancelled by zeros. This is an example of "servo-flutter" or "spill-over."

Optimal Reduced-Order Controllers

Using the numerical algorithm described previously, an optimal second-order controller was found for a sixth-order spacecraft model containing the rigid-body mode and two bending modes. Figure 4 shows the root locus with this controller, which had the transfer function

$$\frac{u(s)}{y(s)} = \frac{0.0489(s+0.273)}{s+0.0291 \pm j0.299} \tag{25}$$

The bandwidth and zero frequency gain of this controller are much lower than those of the controller designed with the rigid spacecraft model. The vibration modes are made only slightly less stable by this second-order controller. This is shown more clearly in Fig. 5, which is the same as Fig. 4 except that the abscissa scale has been magnified. (The controller zero is off the graph.) The black dots are the optimal locations of the closed-loop poles.

The vibration modes are made more stable by higher-order controllers. Figure 6 shows the root locus with a sixth-order controller. The optimal sixth-order controller has the transfer function

$$\frac{u(s)}{y(s)} = \frac{5.28(s+0.114)(s-0.023 \pm j0.838)(s-0.448 \pm j1.34)}{(s+0.039 \pm j.453)(s+0.027 \pm j0.977)(s+2.65 \pm 3.35)} \tag{26}$$

This controller has two right-half plane zeros. Right-half-plane zeros are rarely used in classical compensation networks, but they appear to be common in optimal controllers for systems with poles near the imaginary axis.

This controller was designed by finding the full-order optimal controller for a spacecraft model that had been reduced to sixth-order. Because the performance index with the fourteenth-order spacecraft model and this sixth-order controller is near optimal, the gradient algorithm was not used to improve this controller.

Table 2 Rms values of pitch angle θ, pitch angular velocity $\dot{\theta}$, and gimbal angle u

	Controller order			
	Second	Fourth	Sixth	Fourteenth
θ, μ rad	11.11	6.47	4.58	4.56
$\dot{\theta}$, μ rad/s	14.82	7.73	6.37	6.41
u, μ rad	5.89	7.66	6.83	6.70

Table 3 Performance index $\times 10^6$ vs changes in vibration mode frequencies, $\Delta\omega_i$

	Controller order		
$\Delta\omega_i$, %	2	4	6
−10	4.37	1.81	1.19
−5	4.03	1.67	1.11
0	3.77	1.60	1.08
5	3.57	1.58	1.09
10	3.40	1.60	unstable

Reduced Spacecraft Model

The sixth-order model used to find the controllers, Eqs. (25) and (26), has the transfer function

$$\frac{\theta(s)}{u(s)}=\frac{-6.55\times 10^{-3}\,(s+0.0022\pm j0.44)\,(s+0.0049\pm j0.98)}{s^2\,(s+0.00533\pm j0.871)\,(s+0.0418\pm j2.19)} \tag{27}$$

This model omits the modes where zeros nearly cancel poles as shown in Fig. 1. The constant (-6.55×10^{-3}) results from matching the steady-state response of the model to that of the fourteenth-order model.

Near pole-zero cancellations in transfer functions result from modes being nearly uncontrollable, nearly unobservable, or both. In this example, both the attitude sensor and the ion engines are located on the central body of the spacecraft. Thus, the modes omitted from Eq. (27) are nearly uncontrollable with the engine and nearly unobservable with the sensor.

These modes could be excited by an external source, but their response would have little effect on the sensor output, and a very large control input would be required to shift the eigenvalues of these modes significantly. If vibration in these modes needs to be reduced, another control input and sensor should be added to the system.

Performance of Optimal Controllers

The rms values of θ, $\dot{\theta}$, and u for optimal controllers of several orders are shown in Table 2. Because the disturbance and measurement noise are small, the performance of even the second-order controller is quite good.

The second-order controller [Eq. (25)] has the lowest bandwidth and the lowest rms input value, but the highest rms values of θ and $\dot{\theta}$. The rms values of θ and $\dot{\theta}$ are reduced significantly with the fourth and higher-order optimal controllers.

The full fourteenth-order controller reduces the performance index by less than 2% from that of the sixth-order controller. It has poles and zeros very near the poles and zeros omitted from the sixth-order model.

Sensitivity of Controllers

The sensitivity of the reduced-order controllers was tested by changing all the spacecraft vibration frequencies by up to 10%. The results of this test are shown in Table 3.

The performance of the second-order controller improves as the vibration mode natural frequencies are increased. This results from the vibration modes being removed further from the controller bandwidth and being excited even less by the controller. If the lowest frequency vibration mode were about 30% lower, it would be close enough to the controller bandwidth to result in destabilization.

The fourth-order controller was not sensitive to 10% changes in the vibration-mode frequencies, but the system was unstable with the sixth-order controller when the frequencies were increased by 10% The sixth-order controller is a "notch compensator," i.e., its poles and zeros are very close to the spacecraft poles and zeros; this is the reason it is more sensitive to parameter variations than the second-order controller. It also has higher bandwidth than the second-order controller which, in general, requires a heavier actuator. The full-order controller is probably at least as sensitive to vibration mode frequency variations as the sixth-order controller.

Conclusions

The transfer function from an actuator force input to a collocated position sensor output for an undamped flexible spacecraft has alternating poles and zeros on the imaginary axis in the complex s-plane, and a double pole at $s=0$ (rigid-body mode). A simple lead compensator will stabilize such a system. However, for noisy measurements, the compensator should have at least one more pole than zero.

Low-order controllers designed to minimize a weighted sum of mean square output and mean square input can provide near optimal attitude regulation and are significantly less sensitive to errors in the parameters of interest in modeling flexible spacecraft than are full-order optimal controllers.

The design procedure is considerably more complicated than that of reducing the order of the spacecraft model and designing a full-order optimal controller. However, the extra complication may be worthwhile in view of the adverse spillover effects that can accompany the latter procedure.

Acknowledgments

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